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Particulate mass measurement: a statistical study

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Legislative method of measuring PM

- Particle Mass (PM) declaration is a legislative requirement for vehicle certification in all markets (Europe = 4.5 mg/km).

- PM is measured by capture of a sample of diluted exhaust gas onto one or more glass-fibre filters (TX40).

$$laden - clean = loading$$

Multiple measurements of each filter paper before and after PM loading is statistically beneficial but time consuming.

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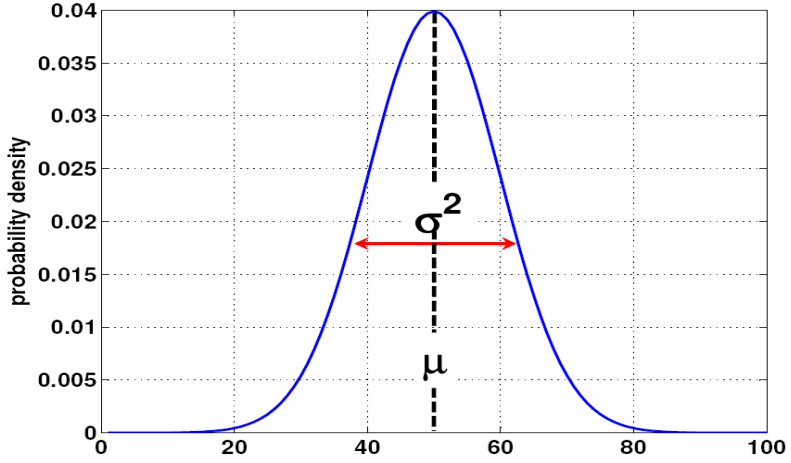
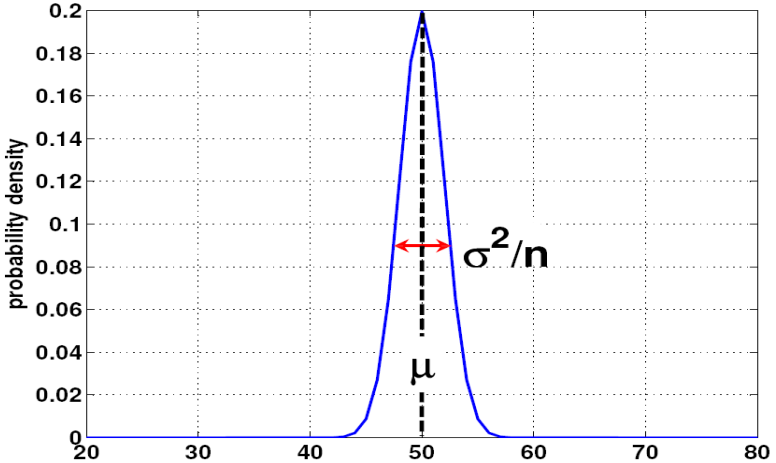
Why are multiple measurements statistically beneficial?

- Whenever a measurement is repeated, the results are never quite identical and variability is observed within a set of repeat readings.

- By taking a larger number of measurements, the sample average is more tightly distributed around the true, but unknown, filter mass.



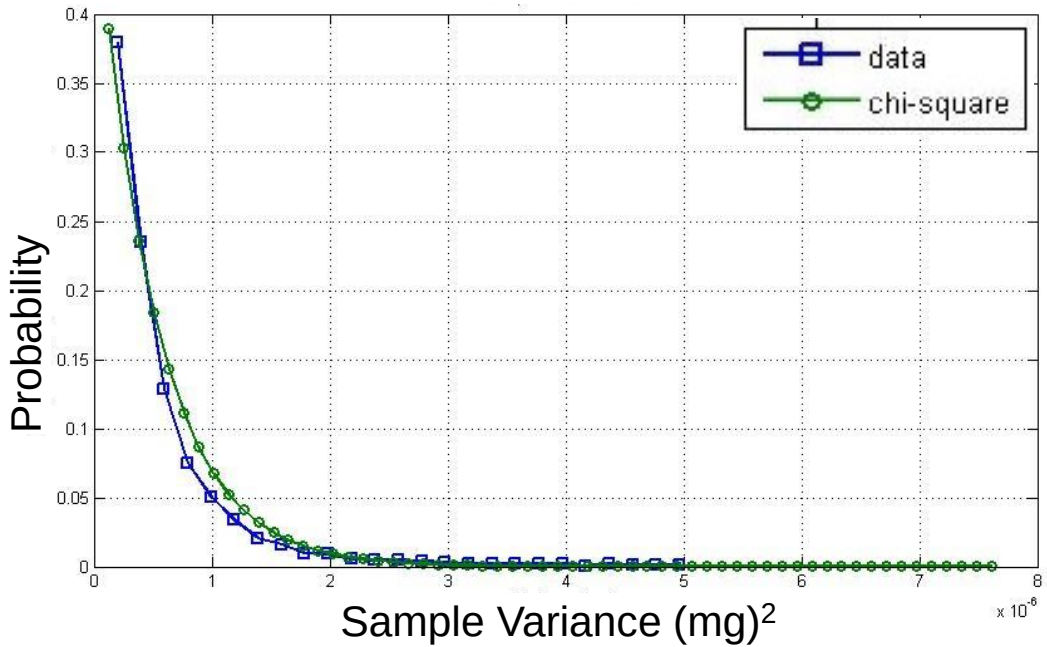
Population distribution



Sample average distribution

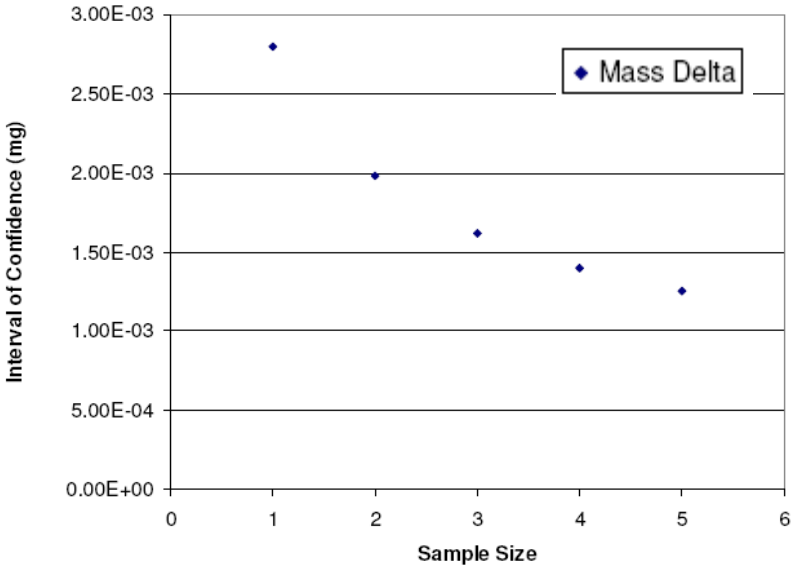
Study of repeatability

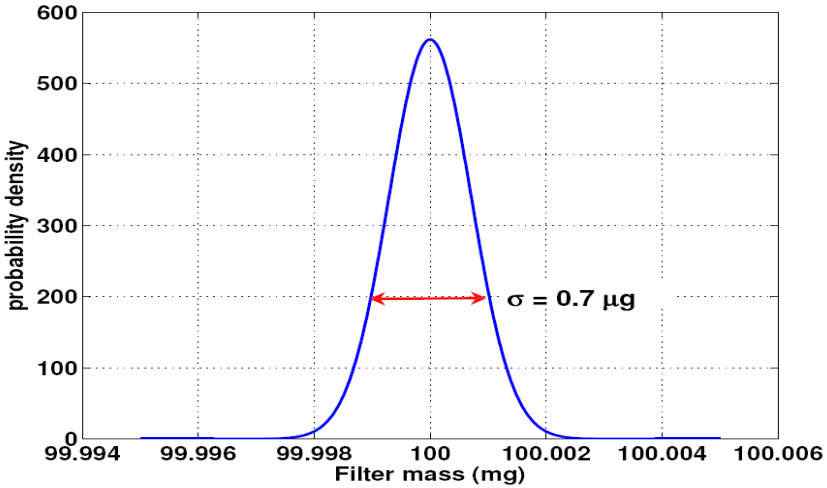
- If sampling from a normal distribution, then the sample variances should be distributed as a Chi-square with 2 degrees of freedom…



Confidence interval v sample size for filter mass measurement

- Based on the data set analysed:
  - $\sigma = 7.1 \times 10^{-4}$ mg





- The uncertainty of PM loading on the filter is twice the variance of the repeatability of the filter measurement.

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Propagation of uncertainty

- The calculated CI's for PM filter loading can be propagated through the European and US legislative equations to assess its contribution to uncertainty in the final result…

$$M_p = \left(\frac{V_{CVS}}{V_{DLS} D} \right) \Delta M$$

$$M_{pi} = \left(\frac{V_{CVS}}{V_{DLS}} \right) \Delta M$$

$$M_p = 0.43 \left(\frac{M_{p1} + M_{p2}}{D_1 + D_2} \right) + 0.57 \left(\frac{M_{p2} + M_{p3}}{D_2 + D_3} \right)$$

ΔM= Change in Mass (mg)

M_p= Total Particulate Mass (mg/km, mg/mile)

M_{pi}= Total Phase Particulate Mass (mg)

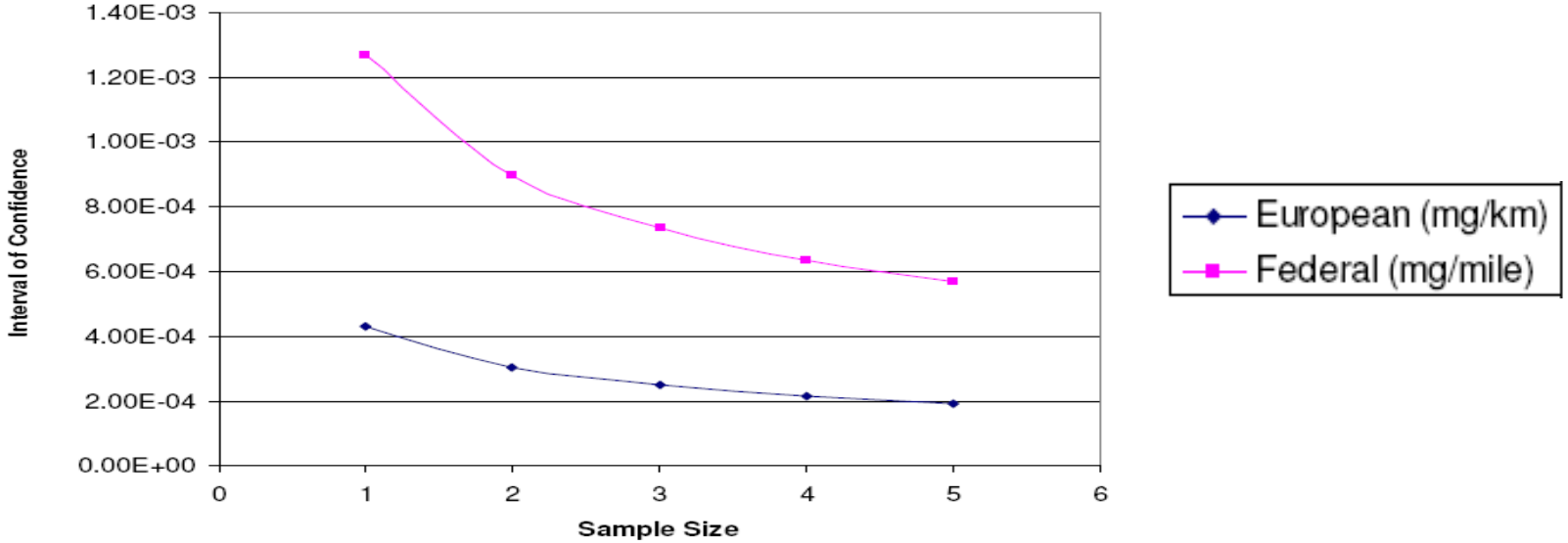
D= Distance Travelled

V_{cvs}= Volume of Gas through Dilution Tunnel

V_{DLS}= Volume of Gas through Filters

Propagation of uncertainty

- The uncertainty in total PM (either mg/km or mg/mile) can be plotted as a function of sample size…



- With an increase in repeat filter mass measurements, the uncertainty in total PM result decreases, but with diminishing return.

Identification of anomalous results

- Even with a sample size n = 1, the 99% confidence interval for the European total PM result is 0.1% of the 4.5mg/km limit.
- However, taking two measurements allows identification of anomalous measurements. If the two repeat measurements differ by more than a predefined tolerance, then one of the results could be anomalous.
- Suggested tolerance of 3.6 µg corresponding to a 99% confidence interval.

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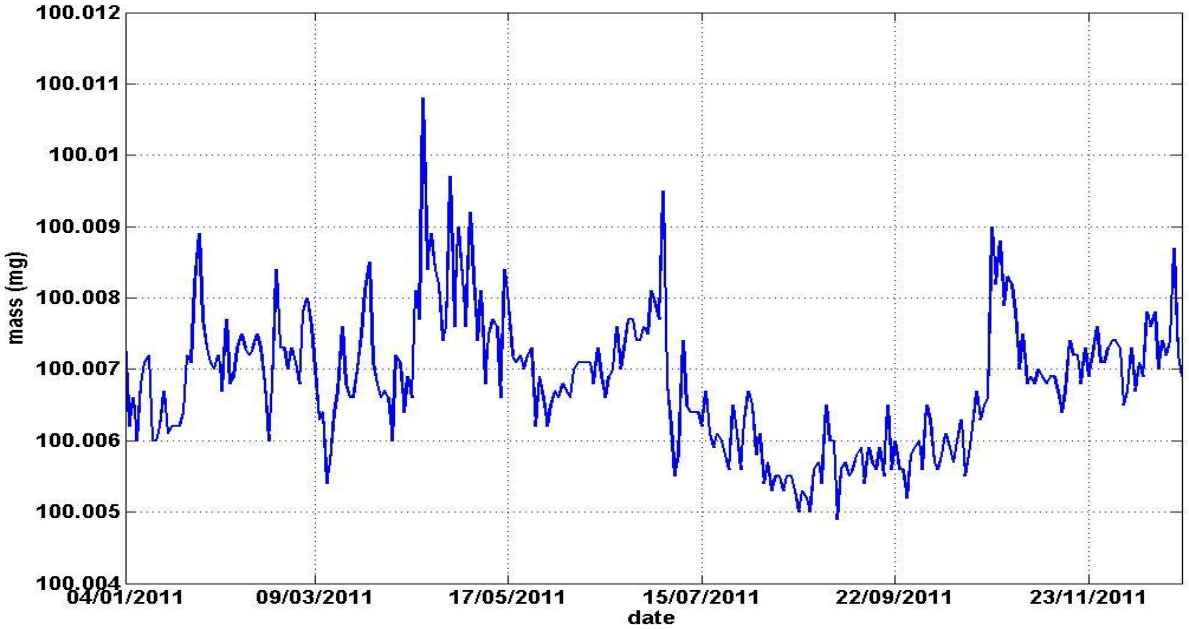
Reference filter mass analysis

- As part of the statistical study, the mass of a reference filter was recorded for three months with an average daily increase of 0.5 µg observed, equating to 0.009 mg/km.

- This result was found to be caused by **moisture uptake** by the filter.

Calibration drift

- Each day, a metal reference weight is measured.
- Range of 5.9 µg, daily average drift of 0.5 µg.



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Summary

A statistical analysis of the particulate mass measurement capability was undertaken and the results support the following conclusions…

- The sample variances followed a Chi-square distribution, indicating that repeatability measurements are normally distributed with σ = 0.7 µg.
- Propagation through the EU and US PM calculations results in a 99% confidence interval of 0.4 µg/km or 1.2 µg/mile respectively.
- Taking two measurements allows for identification of outliers.
- TX40 filters gain mass through moisture uptake at a rate of 0.5 µg/day.
- Apparent drift in balance calibration of 0.5 µg/day.

Thank You for listening. Any questions or comments greatly appreciated.

I would especially welcome any comments on…

The validity of the statistical analysis.

The observed variability in reference weight results.


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